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$\mathcal{N} = 1/2$ gauge theory and its instanton moduli space from open strings in R-R background*

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Abstract: We derive the four dimensional $\mathcal{N} = 1/2$ super Yang-Mills theory from tree-level computations in RNS open string theory with insertions of closed string Ramond-Ramond vertices. We also study instanton configurations in this gauge theory and their ADHM moduli space, using systems of D3 and D(–1) branes in a R-R background.

Keywords: Gauge theories, Instantons, D-branes.

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1. Introduction

The study of the effects of non trivial closed string backgrounds on the low-energy dynamics of open strings and D-branes has attracted a lot of interest in the last few years for many reasons. Among the simplest, yet non-trivial, possibilities that have been considered are the backgrounds in which some of the antisymmetric tensors of the closed string spectrum acquire a constant non-zero value. For example a constant profile for the $B_{\mu\nu}$ field of the NS-NS sector modifies the open string dynamics by introducing new couplings and interactions which can also be interpreted in terms of a non-commutative deformation of the space where the strings propagate [1]. Field theories, and in particular gauge theories, defined on non-commutative spaces were the subject of vast investigations even before the relation with string theory was realized, but it was only after the connection with the propagation of strings in a $B_{\mu\nu}$ background was exhibited that many properties of non-commutative theories were elucidated and put in a broader perspective.

More recently, other kinds of closed string backgrounds have been considered. In particular, in the context of Type II B string theory compactified on a Calabi-Yau threefold, the effects of the presence of a constant non-vanishing graviphoton field strength $C_{\mu\nu}$ have been analyzed by several authors [2, 3, 4, 5, 6]. A graviphoton background can be obtained by wrapping the 5-form field strength of the R-R sector

of Type II B string theory on a 3-cycle of the internal Calabi-Yau manifold, and a consistent possibility in Euclidean space is to take a $C_{\mu\nu}$ with a definite duality, for example anti self-dual. A constant anti self-dual graviphoton field strength induces a deformation of the four dimensional superspace in which the fermionic coordinates are no longer anticommuting Grassmann variables but become elements of a Clifford algebra [7, 8, 2, 3, 4, 5, 6, 9]

$$\left\{\theta^{\alpha},\theta^{\beta}\right\} = \left\{\theta^{\alpha},\bar{\theta}^{\dot{\beta}}\right\} = 0 \quad , \quad \left\{\bar{\theta}^{\dot{\alpha}},\bar{\theta}^{\dot{\beta}}\right\} = C^{\dot{\alpha}\dot{\beta}} \tag{1.1}$$

where $C^{\dot{\alpha}\dot{\beta}} = \frac{1}{4}C_{\mu\nu}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}$. The non-vanishing anticommutator in (1.1) breaks the four dimensional Lorentz group $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ to $\mathrm{SU(2)_L}$, and reduces the number of preserved supercharges by a factor of two. Therefore, a graviphoton background deforms a $\mathcal{N} = 1$ field theory in four dimensions to a $\mathcal{N} = 1/2$ theory with only two preserved supercharges and new types of interactions that are induced by the non-anticommutative structure of the superspace. Supersymmetric field theories based on non-anticommutative superspaces and their renormalization properties have been largely studied in the recent past from different points of view [10, 11, 12, 13, 14, 15, 16]. More recently, also the instanton configurations of the $\mathcal{N} = 1/2$ gauge theory have been analyzed [17, 18, 19] and generalizations with extended supersymmetry have been proposed [20, 21].

Even if the non-anticommutative algebra (1.1) has a direct string theory interpretation as we mentioned above, so far most of the analysis of the $\mathcal{N} = 1/2$ field theories has been carried out by exploiting the superspace deformations that are induced by the graviphoton, without making explicit reference to string theory. In this paper we fill this gap and show that the $\mathcal{N} = 1/2$ gauge theories in four dimensions can be also obtained directly from string theory by computing, in the standard RNS formalism, scattering amplitudes in the presence of a R-R background with constant field strength. It is a common belief that the RNS formalism is not suited to deal with a R-R background; while this is true in general, it is not exactly so when the R-R field strength is constant. In fact, in this case one can represent the background by a R-R vertex operator at *zero* momentum which in principle can be repeatedly inserted inside disk correlation functions among open string vertices without affecting their dynamics. As we will see explicitly in section 2, the integrals on the world-sheet variables that arise from these insertions turn out to be elementary and thus the effects of the R-R background on the open string dynamics can be explicitly computed in this way. Even though this method is intrinsically perturbative, in the field theory limit $\alpha' \to 0$ the procedure stops after the first step and so the results one obtains in this way are exact in this limit. This is a consequence of the fact that the R-R graviphoton background modifies the fermionic sector of the superspace as shown in (1.1) and induces a star product which, when expanded, contains only a finite number of background insertions as a consequence of the fermionic nature of the $\bar{\theta}$'s coordinates.

This paper is organized as follows: in section 2 we briefly review how to engineer the four dimensional $\mathcal{N}=1$ super Yang-Mills theory with gauge group $\mathrm{U}(N)$ in terms of N fractional D3 branes in the orbifold $\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ and, by explicitly computing tree-level scattering amplitudes of open strings on disks with the insertion of a R-R vertex operator, we discuss the deformations induced by a graviphoton background on the world-volume theory. In particular, taking the field theory limit $\alpha'\to 0$ we can recover the action of the $\mathcal{N}=1/2$ super Yang-Mills theory directly from string computations. In section 3 we extend this analysis to a system of N D3 and k D(-1) branes in order to describe the k instanton sector of this gauge theory and, generalizing our previous results [22], we discuss how the structure of the instanton moduli space and the ADHM constraints are modified by the R-R background. This analysis involves the explicit calculation of open string amplitudes on disks which have at least a part of their boundary on the D-instantons and which may also contain insertions of the graviphoton vertices. In section 4 we show that these mixed disks are the sources for the (super)-instantons of the $\mathcal{N}=1/2$ $\mathrm{U}(N)$ gauge theory. In particular, we compute the emission amplitude of the gluon field from a mixed disk in presence of a R-R vertex operator. From this amplitude, in analogy with what happens in the closed string with the boundary state [23], we deduce the leading term in the large distance expansion of the gluon profile in the singular gauge and find how the graviphoton background affects the instanton solution, confirming in this way the general structure that has been recently uncovered in the regular gauge [17, 18, 19]. Finally, in the appendix we list our conventions and collect technical details and useful formulas for our calculations.

2. The $\mathcal{N}=1/2$ gauge theory from open strings in a R-R background

In this section we show how the gauge theory deformations induced by a graviphoton background can be derived directly from string theory. Let us begin by considering the pure $\mathcal{N}=1$ SYM theory in four (euclidean) dimensions with gauge group $\mathrm{U}(N)$ whose action is given by[1]

$$S=\frac{1}{g_{\mathrm{YM}}^2}\int d^4x\,\mathrm{Tr}\Big(\frac{1}{2}F_{\mu\nu}^2-2\bar\Lambda_{\dot\alpha}\bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta\Big)\quad . \tag{2.1}$$

As is well-known this action describes the low-energy dynamics on a stack of N (fractional) D3 branes placed at the singularity of the orbifold $\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$, whose massless excitations are the gauge boson A_μ and the gauginos Λ^α and $\bar\Lambda_{\dot\alpha}$. These are

[1] For our conventions see appendix A.1.

represented by the following open string vertex operators

$$V_A(y;p) = (2\pi\alpha')^{\frac{1}{2}}\,\frac{A_\mu(p)}{\sqrt{2}}\,\psi^\mu(y)\,\mathrm{e}^{-\phi(y)}\,\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \quad , \tag{2.2}$$

$$V_\Lambda(y;p) = (2\pi\alpha')^{\frac{3}{4}}\,\Lambda^\alpha(p)\,S_\alpha(y)\,S^{(-)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\,\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \quad , \tag{2.3}$$

and

$$V_{\bar\Lambda}(y;p) = (2\pi\alpha')^{\frac{3}{4}}\,\bar\Lambda_{\dot\alpha}(p)\,S^{\dot\alpha}(y)\,S^{(+)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\,\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \quad , \tag{2.4}$$

with $p^2 = 0$. In these vertices ϕ is the (chiral) boson of the superghost bosonization formulae [24], X^μ and ψ^μ are the bosonic and fermionic string coordinates along the longitudinal directions of the D3 branes, $S_\alpha\,S^{(-)}$ and $S^{\dot\alpha}\,S^{(+)}$ are the spin field components which survive the GSO and orbifold projections (see appendix A.2, in particular eq. (A.27)), and y is a point on the real axis. Finally, the factors of $(2\pi\alpha')$ in (2.2) – (2.4) have been introduced to assign canonical dimensions to the polarizations, namely $(\text{length})^{-1}$ to the gauge boson and $(\text{length})^{-\frac{3}{2}}$ to the gauginos, keeping, as customary, the vertex operators dimensionless.[2] Note that the above polarizations include also $\mathrm{U}(N)$ Chan-Paton factors T^I in the adjoint representation, which we normalize as

$$\mathrm{Tr}\big(T^I\,T^J\big) = \frac{1}{2}\,\delta^{IJ} \quad . \tag{2.5}$$

The various interaction terms in the super Yang-Mills action (2.1) can be obtained by computing the field theory limit $\alpha' \to 0$ of string scattering amplitudes among the vertex operators (2.2) – (2.4). For example, the (color ordered) amplitude among one gauge boson and two gauginos is

$$\langle\!\langle\, V_{\bar\Lambda}\,V_A\,V_\Lambda\,\rangle\!\rangle \equiv C_4\int\frac{\prod_i dy_i}{dV_{\mathrm{CKG}}}\,\big\langle V_{\bar\Lambda}(y_1;p_1)\,V_A(y_2;p_2)\,V_\Lambda(y_3;p_3)\big\rangle \quad , \tag{2.6}$$

where dV_{CKG} is the $\mathrm{SL}(2,\mathbb{R})$ invariant volume element and C_4 is the topological normalization of a disk with the boundary conditions of a D3 brane given by [25, 22]

$$C_4 = \frac{1}{\pi^2\alpha'^2}\,\frac{1}{g^2_{\mathrm{YM}}} \quad . \tag{2.7}$$

Using the contraction formulas of appendix A.2 and fixing the positions of the vertices to three arbitrary points so that

$$dV_{\mathrm{CGK}} = \frac{dy_a\,dy_b\,dy_c}{(y_a-y_b)(y_b-y_c)(y_c-y_a)} \quad , \tag{2.8}$$

it is easy to find that

$$\langle\!\langle\, V_{\bar\Lambda}\,V_A\,V_\Lambda\,\rangle\!\rangle = -\,\frac{2\,\mathrm{i}}{g^2_{\mathrm{YM}}}\,\mathrm{Tr}\left(\bar\Lambda_{\dot\alpha}(p_1)\,\not{A}^{\dot\alpha\beta}(p_2)\,\Lambda_\beta(p_3)\right) \tag{2.9}$$

[2] Notice that the polarization $A_\mu(p)$ has the *same* dimension of the the field $A_\mu(x)$ because the Fourier transform is taken w.r.t. to the adimensional momentum $k = \sqrt{2\pi\alpha'}p$.

where we have understood the δ-function of momentum conservation (we will do the same also in the following). Note that all factors of α' from the disk normalization C_4 and the vertices cancel out, so that this result survives in the field theory limit. The complete coupling among a gauge boson and two gauginos is obtained by adding to (2.9) the other inequivalent color order of the fields and thus the term $\mathrm{Tr}\big(\bar\Lambda_{\dot\alpha}\big[\bar{\not{A}}^{\dot\alpha\beta},\Lambda_\beta\big]\big)$ of the action (2.1) is recovered. Proceeding systematically in this way, one can check that indeed all interaction terms in (2.1) arise from the $\alpha'\to 0$ limit of scattering amplitudes[3] among the vertices (2.2) – (2.4).

It is interesting to note that the quartic interactions in $\mathrm{Tr}F_{\mu\nu}^2$ can be decoupled by introducing an auxiliary antisymmetric tensor $H_{\mu\nu}$ of definite duality (say, anti self-dual), in the adjoint representation and with dimension (length)$^{-2}$, which we can write as

$$H_{\mu\nu} = H_c\,\bar\eta^c_{\mu\nu} \tag{2.10}$$

where $\bar\eta^c_{\mu\nu}$ are the anti self-dual 't Hooft symbols[4]. In fact the action (2.1) is equivalent to the following one

$$\begin{aligned} S' = \frac{1}{g^2_{\mathrm{YM}}}\int d^4x\ \mathrm{Tr}\Big\{&\big(\partial_\mu A_\nu-\partial_\nu A_\mu\big)\partial^\mu A^\nu\ +\ 2\mathrm{i}\,\partial_\mu A_\nu\big[A^\mu,A^\nu\big]\\ &-\ 2\bar\Lambda_{\dot\alpha}\bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta + H_cH^c + H_c\,\bar\eta^c_{\mu\nu}\big[A^\mu,A^\nu\big]\Big\}\ , \end{aligned} \tag{2.11}$$

which contains only cubic interaction terms. As shown in [22] for the analogous case of D-instantons, also the auxiliary field $H_{\mu\nu}$ of the D3 branes admits a representation in string theory since it can be effectively associated to the following vertex operator (in the 0 superghost picture)

$$V_H(y;p) = (2\pi\alpha')\,\frac{H_{\mu\nu}(p)}{2}\,\psi^\nu\psi^\mu(y)\,\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)}\ , \tag{2.12}$$

which has conformal weight 1 if $p^2=0$. The factor of $(2\pi\alpha')$ has been introduced in order to assign the required dimension to the polarization $H_{\mu\nu}$, which includes also the appropriate $\mathrm{U}(N)$ Chan-Paton factor.

It is very easy to verify that all terms in the action S' can be obtained from the limit $\alpha'\to 0$ of string amplitudes. For example the (color ordered) coupling among the auxiliary field H and two gauge bosons is given by

$$\frac12\langle\!\langle\, V_H\,V_A\,V_A\,\rangle\!\rangle = -\frac{1}{g^2_{\mathrm{YM}}}\,\mathrm{Tr}\,\Big(H_{\mu\nu}(p_1)A^\mu(p_2)A^\nu(p_3)\Big) \tag{2.13}$$

where the symmetry factor of $\frac12$ has been introduced to account for the presence of two alike fields. Again all factors of α' cancel out and this result survives in the field

[3] Remember that in Euclidean space the 1PI part of a scattering amplitude is equal to *minus* the corresponding interaction term in the action.

[4] This choice of duality is related to the fact that later we will introduce an anti self-dual graviphoton background.

theory limit. Adding to (2.13) the amplitude with the other inequivalent color order of the three vertex operators, one reconstructs the last term of (2.11). Furthermore, one can easily check that all other amplitudes involving V_H vanish in the limit $\alpha' \to 0$, so that the complete field theory result is given by the action (2.11).

2.1 The effects of the graviphoton background

We now analyze the deformations of this $\mathcal{N}=1$ gauge theory that are induced by a graviphoton background with constant field strength. This background is usually described by a constant antisymmetric tensor $C_{\mu\nu}$ with definite duality (here we take it to be anti self-dual) which is responsible for a non-anticommutative deformation of the $\mathcal{N}=1$ superspace [2, 3, 4, 5, 6]. From the string point of view $C_{\mu\nu}$ corresponds to a R-R field strength; more precisely it is the R-R 5-form $F^{(5)}$ of type II B string theory, wrapped around the 3-cycle of the internal Calabi-Yau space. In our case the internal space is the orbifold $\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ and the constant graviphoton field strength is described by the following closed string vertex operator (in the $(-1/2,-1/2)$ superghost picture)

$$V_{\mathcal{F}}(z,\bar z) = \mathcal{F}_{\dot\alpha\dot\beta}\, S^{\dot\alpha}(z) S^{(+)}(z) \mathrm{e}^{-\frac{1}{2}\phi(z)}\; \widetilde{S}^{\dot\beta}(\bar z)\widetilde{S}^{(+)}(\bar z)\mathrm{e}^{-\frac{1}{2}\tilde\phi(\bar z)} \tag{2.14}$$

where the dimensionless polarization is a symmetric bi-spinor

$$\mathcal{F}_{\dot\alpha\dot\beta} = \mathcal{F}_{\dot\beta\dot\alpha} \quad . \tag{2.15}$$

In the vertex (2.14) the tilde denotes the right movers, and z a point in the upper-half complex plane. As we will see later, the tensor $C_{\mu\nu}$ that is usually considered in the literature turns out to be proportional to $\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta}$, which is clearly anti self-dual. Notice that the vertex operator (2.14) does not have a $\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X}$ term. In fact, we are considering a *constant* background and hence $p=0$. For this reason, as we shall explicitly see in the following, it is possible to use the RNS formulation of string theory and compute the effects of this R-R background on the gauge theory by evaluating scattering amplitudes on disks with insertions of the vertex operator (2.14) in the interior.

Let us now analyze these mixed open/closed string amplitudes. When the vertex (2.14) is inserted in the interior of a disk, the left and right movers of the closed string become identified as a consequence of the boundary conditions. In the case of a disk representing the world sheet of a D3 brane, the relevant boundary conditions for the spin fields are (see, for example, eq. (2.5) of Ref. [22])

$$S^{\dot\alpha}(z)\, S^{(+)}(z) = \widetilde{S}^{\dot\alpha}(\bar z)\, \widetilde{S}^{(+)}(\bar z)\Big|_{z=\bar z} \ , \tag{2.16}$$

having conformally mapped the disk to the upper half plane and hence its boundary to the real axis. The calculation of a disk amplitude with the insertion of the closed

Figure 1: D3 disk amplitudes involving the R-R background and the gauge field A_μ *(a)*, the auxiliary field $H_{\mu\nu}$ *(b)*.

string vertex (2.14) is then performed by replacing in the latter the right moving spin fields with the left moving ones, according to

$$\widetilde{S}^{\dot\alpha}(\bar z)\,\widetilde{S}^{(+)}(\bar z) \;\longrightarrow\; S^{\dot\alpha}(\bar z)\,S^{(+)}(\bar z) \quad . \tag{2.17}$$

Because of this replacement, any insertion of the R-R vertex (2.14) will introduce two internal spin fields of type $S^{(+)}$ whose "charge" has to be compensated by two internal spin fields of type $S^{(-)}$ in order to have a non-vanishing amplitude. The only vertex that contains $S^{(-)}$ is that of the gaugino Λ (see eq. (2.3)), and thus we easily conclude that any insertion of the graviphoton field strength $\mathcal{F}_{\dot\alpha\dot\beta}$ must be accompanied by two gauginos Λ^α and Λ^β. However, due to the different chiralities involved and the symmetry properties of $\mathcal{F}_{\dot\alpha\dot\beta}$, it is immediate to realize that some other field is necessary in order to saturate the spinor indices and produce a non-zero result. Indeed, with only one $V_{\mathcal{F}}$ and two V_Λ's, the correlator among the SO(4) spin fields is proportional to $\epsilon_{\alpha\beta}\,\epsilon^{\dot\alpha\dot\beta}$ (see eq. (A.24) in appendix A.2) which vanishes when contracted with the symmetric bi-spinor $\mathcal{F}_{\dot\alpha\dot\beta}$. The simplest possibility to avoid this is to insert a gluon vertex V_A, and thus consider the following amplitude

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}}\,\rangle\!\rangle \equiv C_4\int\frac{\prod_i dy_i dz d\bar z}{dV_{\rm CKG}}\;\Big\langle V_\Lambda(y_1;p_1)\,V_\Lambda(y_2;p_2)\,V_A(y_3;p_3)\,V_{\mathcal{F}}(z,\bar z)\Big\rangle \tag{2.18}$$

which is represented in Figure 1a. Note that the vertices of the two gauginos and of the graviphoton background already saturate the superghost charge anomaly, and thus in (2.18) the vertex V_A must be taken in the 0 superghost picture. In this picture, the properly normalized integrated gluon vertex is (up to ghost terms) [22]

$$V_A(y;p) = 2\mathrm{i}\,(2\pi\alpha')^{\frac12}\,A_\mu(p)\,\Big(\partial X^\mu(y) + \mathrm{i}\,(2\pi\alpha')^{\frac12}\,p\cdot\psi\,\psi^\mu(y)\Big)\,\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X(y)} \tag{2.19}$$

but, for the reasons explained above, only the $p\cdot\psi\,\psi^\mu$ part can contribute. The

amplitude (2.18) then becomes

$$\begin{aligned}\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}}\,\rangle\!\rangle = &\frac{8}{g^2_{\rm YM}}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda^\alpha(p_1)\,\Lambda^\beta(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}\\ &\times\int\frac{\prod_i dy_i dz d\bar z}{dV_{\rm CKG}}\,\Bigg\{\big\langle S_\alpha(y_1)\,S_\beta(y_2)\;:\psi^\nu\psi^\mu:(y_3)\,S^{\dot\alpha}(z)\,S^{\dot\beta}(\bar z)\big\rangle\\ &\times\big\langle S^{(-)}(y_1)\,S^{(-)}(y_2)\,S^{(+)}(z)\,S^{(+)}(\bar z)\big\rangle\,\big\langle {\rm e}^{-\frac{1}{2}\phi(y_1)}{\rm e}^{-\frac{1}{2}\phi(y_2)}{\rm e}^{-\frac{1}{2}\phi(z)}{\rm e}^{-\frac{1}{2}\phi(\bar z)}\big\rangle\\ &\times\big\langle {\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}{\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}{\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\big\rangle\Bigg\}\quad .\end{aligned}\tag{2.20}$$

We now use the correlation functions given in appendix A.2 and exploit the $\mathrm{SL}(2,\mathbb{R})$ invariance to fix $y_1\to\infty$, $z\to{\rm i}$ and $\bar z\to -{\rm i}$, so that we are left to perform the following integral[5]:

$$\int_{-\infty}^{+\infty}dy_2\int_{-\infty}^{y_2}dy_3\,\frac{1}{(y_2^2+1)\,(y_3^2+1)}=\frac{\pi^2}{2}\quad .\tag{2.21}$$

Collecting all terms, in the end we find

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}}\,\rangle\!\rangle = \frac{8\pi^2}{g^2_{\rm YM}}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda(p_1)\!\cdot\!\Lambda(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta}\quad .\tag{2.22}$$

The complete coupling is obtained by multiplying this result by a symmetry factor of $\frac{1}{2}$ to account for the two alike gauginos and then by adding to it the amplitude corresponding to the other inequivalent color order of the three open string vertex operators; however, these two effects compensate each other and so the right hand side of (2.22) is the full answer. From this we clearly see that the field theory limit $\alpha'\to 0$ yields a trivial result unless we rescale the graviphoton field strength $\mathcal{F}_{\dot\alpha\dot\beta}$ to infinity, in such a way that the following combination

$$4\pi^2\,(2\pi\alpha')^{\frac{1}{2}}\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta}\equiv C_{\mu\nu}\tag{2.23}$$

which has dimensions of a (length), remains constant. If we do this, then the amplitude (2.22) survives in the field theory limit and produces the following term in the gauge theory action

$$\frac{\rm i}{g^2_{\rm YM}}\int d^4x\;{\rm Tr}\,\Big(\Lambda\!\cdot\!\Lambda\big(\partial^\mu A^\nu-\partial^\nu A^\mu\big)\Big)C_{\mu\nu}\quad .\tag{2.24}$$

[5]It is also possible to fix the $\mathrm{SL}(2,\mathbb{R})$ symmetry in a more conventional way by choosing $y_1=\infty$, $y_2=1$ and $y_3=0$ and in this way to obtain the integral $\int_{z\in H^+}dzd\bar z\frac{2{\rm i}y}{|z|^2\,|1-z|^2}$ over the position of the closed string emission vertex $z=x+{\rm i}y$ in the upper half plane. However this integral, as it stands, has a logarithmic divergence for $z\to 1$; this can be cured by introducing a cutoff $y>\epsilon$ and letting it go to zero at the end of the computation. The result we obtain is the same as using the other gauge fixing. The reason of such a procedure is to avoid that the closed string emission vertex collides with the border, condition which is automatically implemented by gauge fixing $z={\rm i}$.

Since the gluon vertex operator (2.19) has the same fermionic structure as the auxiliary vertex (2.12), we should consider also the amplitude $\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_H\, V_{\mathcal{F}} \,\rangle\!\rangle$, depicted in Figure 1$b$, whose evaluation follows exactly the same steps we have just described. In this case we have

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_H\, V_{\mathcal{F}} \,\rangle\!\rangle = \frac{2\pi^2}{g_{\rm YM}^2}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda(p_1)\cdot\Lambda(p_2)\,H^{\mu\nu}(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta} \quad . \tag{2.25}$$

Thus, after using (2.23), we conclude that also the following term must be added to the gauge theory action

$$\frac{1}{2g_{\rm YM}^2}\int d^4x\;{\rm Tr}\,\Big(\Lambda\cdot\Lambda\,H^{\mu\nu}\Big)C_{\mu\nu} \quad . \tag{2.26}$$

It is worth pointing out that the disk amplitudes (2.18) and (2.25) correspond to 5-point correlation functions from the two-dimensional world-sheet point of view, since the closed string vertex $V_{\mathcal{F}}$ effectively counts as two open string vertices due to the reflection rules (2.17). However, the same amplitudes correspond to 3-point functions from the point of view of the D3 brane world-volume, since there are only three vertex operators (those associated to the massless excitations of the open strings) which carry momentum in four dimensions and represent dynamical degrees of freedom.

It is not difficult to verify that any other disk amplitude with more insertions of the R-R vertex operator (2.14), either is zero because of index structure, or vanishes in the field theory limit if the combination (2.23) is kept fixed. Thus, even if we are treating the closed string background in a perturbative way by means of successive insertions of vertices $V_{\mathcal{F}}$, in our case this perturbative procedure terminates after the first step. The terms (2.24) and (2.26) are then the only two modifications produced by the graviphoton background in the $\alpha'\to 0$ limit on the gauge theory action of N D3 branes, which then becomes

$$\begin{aligned}\widetilde{S}' = \frac{1}{g_{\rm YM}^2}\int d^4x\;{\rm Tr}\Big\{&(\partial_\mu A_\nu-\partial_\nu A_\mu)\partial^\mu A^\nu + 2{\rm i}\,\partial_\mu A_\nu\big[A^\mu,A^\nu\big] - 2\bar\Lambda_{\dot\alpha}\bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta \\ &+{\rm i}\big(\partial^\mu A^\nu-\partial^\nu A^\mu\big)\Lambda\cdot\Lambda\,C_{\mu\nu} + H_cH^c + H_c\,\bar\eta^c_{\mu\nu}\Big(\big[A^\mu,A^\nu\big]+\frac{1}{2}\,\Lambda\cdot\Lambda\,C^{\mu\nu}\Big)\Big\} \quad .\end{aligned} \tag{2.27}$$

Integrating out the auxiliary field H, we finally get

$$\begin{aligned}\widetilde{S} &= \frac{1}{g_{\rm YM}^2}\int d^4x\;{\rm Tr}\Big\{\frac{1}{2}F_{\mu\nu}^2 - 2\bar\Lambda_{\dot\alpha}\bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta + {\rm i}\,F^{\mu\nu}\,\Lambda\cdot\Lambda\,C_{\mu\nu} - \frac{1}{4}\Big(\Lambda\cdot\Lambda\,C_{\mu\nu}\Big)^2\Big\} \\ &= \frac{1}{g_{\rm YM}^2}\int d^4x\;{\rm Tr}\,\Big\{\Big(F^{(-)}_{\mu\nu}+\frac{\rm i}{2}\,\Lambda\cdot\Lambda\,C_{\mu\nu}\Big)^2 + \frac{1}{2}F_{\mu\nu}\widetilde{F}^{\mu\nu} - 2\bar\Lambda_{\dot\alpha}\bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta\Big\}\end{aligned} \tag{2.28}$$

which, in our conventions (see appendix A.1), exactly agrees with the $\mathcal{N}=1/2$ action of [5]. Therefore, we have shown that the C-interactions of the $\mathcal{N}=1/2$ super Yang-Mills theory, which are usually derived from a non-anticommutative deformation of the superspace, can also be obtained directly from string theory, and in particular from the $\alpha' \to 0$ limit of open string scattering amplitudes in the presence of R-R vertex operators together with an appropriate rescaling of the graviphoton field strength.

3. ADHM instanton moduli space in the $\mathcal{N}=1/2$ theory

In this section we describe how the R-R background that deforms the world-volume dynamics on the D3 branes leading to the $\mathcal{N}=1/2$ gauge theory, also modifies the moduli space of its (super)-instantons. Instantons represent an intrinsically non-perturbative feature of a gauge theory; nevertheless, many aspects of their physics can be reproduced by perturbative open string computations on systems of D3 branes *and* D-instantons [28, 29, 27, 22]. In this framework, we show that the effects of the graviphoton background on the D-instantons can be taken into account in a very similar way to what we did in the previous section for the D3 branes.

3.1 The undeformed moduli space

The moduli space of the (super)-instanton solutions of $\mathrm{U}(N)$ (super)-Yang-Mills theory is described by the ADHM construction [26]. This construction can be naturally recast in a stringy language (for a review see, for instance, [27] and references therein); in fact, for instanton number k, one simply adds k D-instantons to the N D3 branes on which the gauge theory lives. The auxiliary variables appearing in the ADHM construction correspond to the degrees of freedom of open strings with at least one end-point attached to a D-instanton. In [22] we presented in detail the derivation of the action for the instanton moduli starting from open string disk amplitudes in flat space, corresponding to $\mathcal{N}=4$ gauge theory. Here we briefly review the basic steps of this derivation, adapting it to the $\mathcal{N}=1$ case with target space $\mathbb{R}^{1,3}\times(\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2))$ which is relevant for our further developments.

In section 2, we saw that the tree-level gauge theory action (2.1) arises from open string amplitudes computed on disks whose boundaries lie entirely on the D3 branes, and evaluated in the limit $\alpha' \to 0$ with the coupling g_{YM} and the dimensionful fields A_μ, Λ^α and $\bar\Lambda_{\dot\alpha}$ kept constant. The moduli action and the ADHM constraints arise instead from open string amplitudes computed on disks with at least part of their boundaries on the D-instantons. However, the coupling constant g_0 which naturally appears in the "gauge theory" on the D-instantons is not independent from g_{YM}; the relation between the two is summarized by writing the normalization C_0 of disks

attached to D-instantons [22, 25]

$$C_0 = \frac{1}{2\pi^2\alpha'^2}\,\frac{1}{g_0^2} = \frac{8\pi^2}{g_{\rm YM}^2} \quad . \tag{3.1}$$

Clearly, if $g_{\rm YM}$ is kept fixed when $\alpha' \to 0$, then g_0, which has dimensions of (length)$^{-2}$, must blow up. This entails the fact that the moduli have to be rescaled with appropriate powers of g_0 to retain some non-trivial interactions in the field theory limit [27, 22]. In this way, the moduli acquire the dimensions which are appropriate for their interpretation as parameters of an instanton solution. For instance, in the NS sector of the D(–1)/D(–1) strings one would naturally define the "massless" vertex operator

$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\,\frac{a_\mu}{\sqrt{2}}\;\psi^\mu(y)\,{\rm e}^{-\phi(z)} \quad , \tag{3.2}$$

where the moduli a_μ have dimensions of (length)$^{-1}$, just as the gluon field A_μ of the D3/D3 strings. However, in order to have non-vanishing disk amplitudes in the $\alpha' \to 0$ limit taken as mentioned above, one must keep fixed the rescaled moduli [22]

$$a'_\mu = \frac{1}{\sqrt{2}g_0} a_\mu \quad , \tag{3.3}$$

which have dimensions of (length) and are related to the position(s) of the (multi)-centers of the instanton solution. Note that the above moduli carry also Chan-Paton factors t^U in the adjoint of U(k), which are normalized as

$$\mathrm{tr}\big(t^U\,t^V\big) = \delta^{UV} \quad . \tag{3.4}$$

In the R sector of the D(–1)/D(–1) strings on the orbifold, we have four fermionic moduli M'^α and $\lambda^{\dot\alpha}$ which are associated to the vertices

$$\begin{aligned} V_M(y) &= (2\pi\alpha')^{\frac{3}{4}}\,\frac{g_0}{\sqrt{2}}\,M'^\alpha\,S_\alpha(y)S^{(-)}(y)\,{\rm e}^{-\frac{1}{2}\phi(y)} \quad , \\ V_\lambda(y) &= (2\pi\alpha')^{\frac{3}{4}}\,\lambda'_{\dot\alpha}\,S^{\dot\alpha}(y)S^{(+)}(y)\,{\rm e}^{-\frac{1}{2}\phi(y)} \quad , \end{aligned} \tag{3.5}$$

where we have already taken into account the rescalings that are suitable to the $\alpha' \to 0$ limit [22]. Thus, M'^α has dimensions of (length)$^{\frac{1}{2}}$, while $\lambda'_{\dot\alpha}$ retains dimensions of (length)$^{-\frac{3}{2}}$. Also these moduli have Chan-Paton factors in the adjoint of U(k).

Let us now consider the strings that are stretched between a D3 and a D(–1) brane. They are characterized by the fact that the four longitudinal directions to the D3 branes have mixed boundary conditions. Thus, in the NS sector of the D3/D(–1) and D(–1)/D3 strings find the following physical vertices

$$\begin{aligned} V_w(y) &= (2\pi\alpha')^{\frac{1}{2}}\,\frac{g_0}{\sqrt{2}}\,w'_{\dot\alpha}\,\Delta(y)\,S^{\dot\alpha}(y)\,{\rm e}^{-\phi(y)} \quad , \\ V_{\bar w}(y) &= (2\pi\alpha')^{\frac{1}{2}}\,\frac{g_0}{\sqrt{2}}\,\bar w'_{\dot\alpha}\,\bar\Delta(y)\,S^{\dot\alpha}(y)\,{\rm e}^{-\phi(y)} \quad , \end{aligned} \tag{3.6}$$

where Δ and $\bar{\Delta}$ are twist operators of conformal weight $1/4$ (we refer to appendix A.2 for their definition and some of their properties). The bosonic moduli $w'_{\dot\alpha}$ and $\bar{w}'_{\dot\alpha}$ carry Chan-Paton factors, respectively, in the bifundamental representations $\mathbf{N} \times \mathbf{k}$ and $\bar{\mathbf{N}} \times \bar{\mathbf{k}}$ of the gauge groups and therefore one should write more explicitly ${w'}^{iu}{}_{\dot\alpha}$ and $\bar{w}'_{\dot\alpha ui}$, where $u = 1, \ldots, N$ and $i = 1, \ldots, k$. As one can see from (3.6), w' and w' have dimensions of a (length) and are in fact related to the size of the instanton solution.

Finally, in the R sector of the D3/D(–1) and D(–1)/D3 strings, we find the vertices

$$
\begin{aligned}
V_\mu(y) &= (2\pi\alpha')^{\frac{3}{4}} \,\frac{g_0}{\sqrt{2}}\, \mu' \,\Delta(y)\, S^{(-)}(y)\, \mathrm{e}^{-\frac{1}{2}\phi(y)} \quad , \\
V_{\bar\mu}(y) &= (2\pi\alpha')^{\frac{3}{4}} \,\frac{g_0}{\sqrt{2}}\, \bar\mu' \,\bar\Delta(y)\, S^{(-)}(y)\, \mathrm{e}^{-\frac{1}{2}\phi(y)} \quad .
\end{aligned}
\tag{3.7}
$$

The fermionic moduli μ' and $\bar\mu'$ have dimensions of $(\text{length})^{1/2}$, and carry the same Chan-Paton factors as the w''s and $\bar{w}'$'s. From now on, to simplify a bit the notation, we will drop the primes from all rescaled moduli, except from a' and M' for which they are traditional in the literature.

The vertices (3.2), (3.5), (3.6) and (3.7) exhaust the BRST-invariant spectrum of the open strings with at least one end point on the D-instantons. However, in order to compute the quartic interactions among the moduli, it is necessary to introduce *auxiliary moduli* [22], which are the strict analogue of the auxiliary fields $H_{\mu\nu}$ we introduced in section 2 for the D3/D3 gauge theory. These new auxiliary moduli disentangle the quartic interactions, so that the moduli action has only cubic terms. The relevant auxiliary vertex operator that survives the orbifold projection is

$$
V_D(y) = (2\pi\alpha')\,\frac{D_c\,\bar\eta^c_{\mu\nu}}{2}\,\psi^\nu\psi^\mu(y) \quad , \tag{3.8}
$$

and describes an excitation of the D(–1)/D(–1) strings. Note that this vertex is in the 0-superghost picture and that its polarization has been rescaled according to our general rules [22].

Computing all cubic tree-level interactions among the vertices listed above and taking the field theory limit (with $g_0 \to \infty$) we obtain the following action for the instanton moduli of the $\mathcal{N} = 1$ super Yang-Mills theory

$$
S_{\rm mod} = \mathrm{tr}\Big\{ -\,\mathrm{i}D_c\Big(W^c + \mathrm{i}\bar\eta^c_{\mu\nu}\big[{a'}^\mu, {a'}^\nu\big]\Big) - \mathrm{i}\lambda^{\dot\alpha}\Big(w^u{}_{\dot\alpha}\,\bar\mu_u + \mu^u \bar{w}_{\dot\alpha u} + \big[a'_{\alpha\dot\alpha}, {M'}^\alpha\big]\Big)\Big\} \tag{3.9}
$$

where we introduced the $k \times k$ matrices

$$
(W^c)_j{}^i = w^{iu}{}_{\dot\alpha}\,(\tau^c)^{\dot\alpha}{}_{\dot\beta}\,\bar{w}^{\dot\beta}{}_{uj} \tag{3.10}
$$

with τ^c being the Pauli matrices, and indicated explicitly the trace over the U(k) indices $i, j, \ldots$.

The moduli action (3.9) is much simpler than the corresponding one for the $\mathcal{N} = 4$ theory (see for instance [27]) and only accounts for the ADHM constraints without any further structure. In fact, the moduli D_c and $\lambda^{\dot{\alpha}}$ appear as Lagrange multipliers, respectively, for the bosonic ADHM constraints, which are the following three $k \times k$ matrix equations

$$W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = \mathbf{0} \quad , \tag{3.11}$$

and for their fermionic counterparts

$$w^u{}_{\dot{\alpha}}\,\bar{\mu}_u + \mu^u \bar{w}_{\dot{\alpha}u} + \left[a'_{\alpha\dot{\alpha}}, M'^{\alpha}\right] = \mathbf{0} \quad . \tag{3.12}$$

Once these constraints are satisfied, the moduli action (3.9) vanishes.

3.2 The R-R deformation of the moduli space

Now we want to take into account the effect on the instanton moduli space of the closed string R-R background that we introduced in section 2. To do so we must compute amplitudes on disks which have at least part of their boundary on the D-instantons, have some insertions of moduli vertices on the boundary and also some insertions of the R-R vertex operator (2.14) in the interior of the disk . In computing these mixed open/closed string amplitudes we must properly take into account the reflection rules associated to the D(–1) boundary, which relate the anti-holomorphic to the holomorphic part of the closed vertex operators. It turns out (see, for example, eq. (2.4) of Ref. [22]) that on a D(–1) boundary the spin fields appearing in the R-R vertex operator (2.14) have exactly the same reflection properties of a D3 boundary given in (2.16). Thus also for the amplitudes we are now considering, we can replace the right moving parts of the spin fields in the graviphoton vertex with the left moving ones according to the rule (2.17).

Let us first consider disks whose boundary lies entirely on the D(–1) branes; in other words we insert no boundary changing moduli w, $\bar{w}$, μ or $\bar{\mu}$, and hence no twist operators Δ or $\bar{\Delta}$. The situation is then strictly analogous to that of the D3 disks we considered in section 2. Following the same reasoning given after (2.17), once a R-R vertex $V_{\mathcal{F}}$ is inserted inside a correlator, we must insert also two fermionic vertex operators V_M in order to balance the "charge" of the internal spin fields, and one auxiliary vertex V_D in order to properly saturate the spinor indices and get a non-vanishing result. Thus, we must compute the amplitude

$$\langle\!\langle\, V_M V_M V_D V_{\mathcal{F}} \,\rangle\!\rangle \tag{3.13}$$

which corresponds to the diagram depicted in of Figure 2a. The computation of this amplitude follows exactly the same steps described for the amplitudes (2.18) and (2.25) in section 2. Taking into account the disk normalization C_0 given in (3.1) and

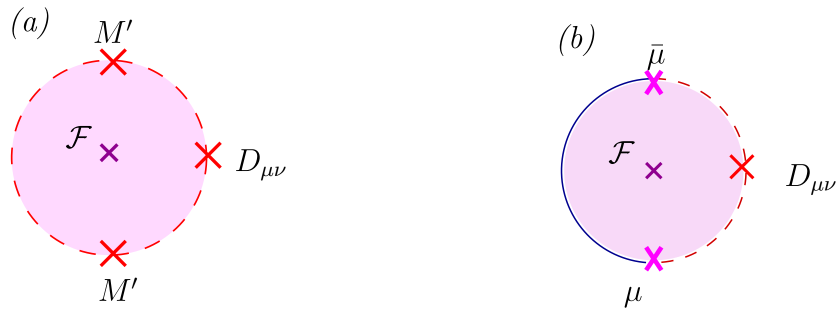

Figure 2: Non-zero diagrams with R-R insertions on a D(–1) disk *(a)* and on a mixed disk *(b)*.

the explicit expressions of the relevant vertices with their proper normalizations, we find

$$\begin{aligned} \langle\!\langle\, V_M V_M V_D V_{\mathcal{F}} \,\rangle\!\rangle &= \frac{\pi^2}{2}\,(2\pi\alpha')^{\frac{1}{2}}\,\mathrm{tr}\Big(M'\cdot M' D_c\Big)\bar{\eta}^c_{\mu\nu}\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar{\sigma}_{\nu\mu})^{\dot\alpha\dot\beta} \\ &= -\frac{1}{2}\,\mathrm{tr}\Big(M'\cdot M' D_c\Big)C^c\ , \end{aligned} \tag{3.14}$$

where we have defined

$$C^c = \frac{1}{4}\bar{\eta}^c_{\mu\nu}\,C^{\mu\nu} \tag{3.15}$$

with $C^{\mu\nu}$ being the rescaled graviphoton field-strength introduced in (2.23).

In the D3/D(–1) system there is also another non-vanishing amplitude involving the graviphoton background. Indeed, we can balance the "charge" of the internal spin fields of the R-R vertex $V_{\mathcal{F}}$ also with a pair of boundary changing operators V_μ and $V_{\bar\mu}$, so that we should also consider the amplitude

$$\langle\!\langle\, V_{\bar\mu} V_\mu V_D V_{\mathcal{F}} \,\rangle\!\rangle \tag{3.16}$$

which corresponds to the *mixed* disk represented in Figure 2*b*. At first sight, the evaluation of this mixed amplitude seems rather involved because the disk has two types of boundary and hence two types of boundary reflection rules should be implemented on the closed string vertex operator. However, as we already mentioned, the spin fields that appear in the graviphoton vertex (2.14) have the *same* boundary conditions on both kinds of boundaries [22], and so also for mixed disks the reflection properties are those of (2.17). The amplitude (3.16) can then be evaluated following the same steps described above and using, as specific ingredients, the correlator of two bosonic twist fields given in (A.29) and the SO(4) correlator among a current and two spin fields given in (A.25). Taking into account all normalization factors, in the field theory limit we finally find

$$\begin{aligned} \langle\!\langle\, V_{\bar\mu} V_\mu V_D V_{\mathcal{F}} \,\rangle\!\rangle &= \frac{\pi^2}{2}\,(2\pi\alpha')^{\frac{1}{2}}\,\mathrm{tr}\Big(\bar{\mu}_u\mu^u D_c\Big)\bar{\eta}^c_{\mu\nu}\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar{\sigma}^{\nu\mu})^{\dot\alpha\dot\beta} \\ &= -\frac{1}{2}\,\mathrm{tr}\Big(\bar{\mu}_u\mu^u D_c\Big)C^c\ . \end{aligned} \tag{3.17}$$

With a systematic analysis one can show that there are no other non-vanishing diagrams on D(–1) or mixed disks involving the graviphoton background which survive the $\alpha' \to 0$ limit, and thus (3.14) and (3.17) are the only terms that modify the moduli action $S_{\rm mod}$. Varying such a deformed action with respect to the auxiliary fields D_c, we obtain the modified ADHM bosonic constraints, which we again write as three $k \times k$ matrix equations

$$W^c + {\rm i}\bar\eta^c_{\mu\nu}\big[a'^\mu, a'^\nu\big] + \frac{\rm i}{2}\Big(M'\!\cdot\! M' - \mu^u\bar\mu_u\Big)C^c = \mathbf{0} \quad . \tag{3.18}$$

Since there are no new types of interactions involving the fermionic moduli $\lambda^{\dot\alpha}$ and the graviphoton background, the fermionic ADHM constraints (3.12) remain unchanged.

We conclude this section by mentioning that a similar analysis can also be performed to describe the moduli space of anti-instantons (*i.e.* gauge configurations with anti self-dual field strength). In this case, however, one has to reverse the GSO projections on the vertex operators of the moduli w, $\bar w$, M' and λ, which then acquire an opposite SO(4) chirality as compared to what we had before, and use an auxiliary vertex V_D as in (3.8) but with $\bar\eta^c_{\mu\nu}$ replaced by $\eta^c_{\mu\nu}$. As a consequence of these changes, any string amplitude involving the anti self-dual R-R field strength will vanish since the relevant quantity C^c becomes proportional to $\eta^c_{\mu\nu}\, C^{\mu\nu}$ which is zero. Thus, in the case of anti-instantons the ADHM constraints are not modified by the anti self-dual graviphoton background; this result is also in agreement with the structure of the anti-instanton solutions recently found in [17, 18, 19].

4. The profile of the deformed instanton solutions

We now study the instanton solutions of the $\mathcal{N} = 1/2$ gauge theory and analyze how the R-R background affects them. We adopt the same strategy described in detail in [22] where we have shown that the mixed disks of the D3/D(–1) system are the sources for the classical (super)-instanton solution. In fact, by computing the emission amplitude for the gauge vector multiplet from a mixed disk and taking its Fourier transform after inserting a free propagator, one obtains the leading term in the large distance expansion of the (super)-instanton solution in the singular gauge [22]. For simplicity, but without loss in generality, here we discuss only the case of instanton number $k = 1$.

Let us begin with the U(N) gauge field A^I_μ. There are two mixed disk diagrams that contribute to the gluon emission and they are represented in Figures 3a and 3b. The first diagram does not involve the R-R background and corresponds to the following amplitude

$$\langle\!\langle\, V_{\bar w}\, \mathcal{V}_{A^I_\mu}(-p)\, V_w \,\rangle\!\rangle \tag{4.1}$$

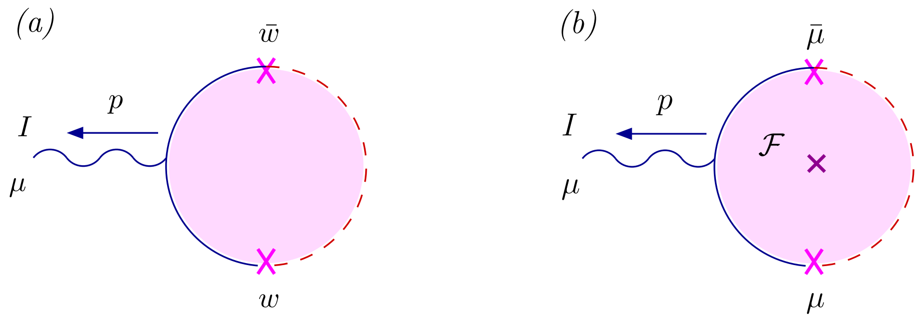

Figure 3: Mixed disks that describe the emission of a gauge vector field A^I_μ with momentum p and without a R-R insertion *(a)* or with a R-R insertion *(b)*.

where in the gluon vertex we have removed the polarization and put an *outgoing* momentum in such a way that the result has the Lorentz structure and the quantum numbers of an emitted gauge vector field. Thus, the gluon vertex operator that we must use in (4.1) is (in the 0 superghost picture)

$$\mathcal{V}_{A^I_\mu}(-p) = 2\mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\left(\partial X^\mu - \mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\,p\cdot\psi\,\psi^\mu\right)\mathrm{e}^{-\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X} \quad . \tag{4.2}$$

As in other amplitudes we have considered before, only the $p{\cdot}\psi\,\psi^\mu$ term contributes in the correlation (4.1); performing the calculation we find, as in [22],

$$\langle\!\langle\, V_{\bar{w}}\,\mathcal{V}_{A^I_\mu}(-p)\,V_w\,\rangle\!\rangle = \mathrm{i}\,(T^I)^v{}_u\,p^\nu\,\bar{\eta}^c_{\nu\mu}\big(w^u{}_{\dot{\alpha}}\,(\tau^c)^{\dot{\alpha}}{}_{\dot{\beta}}\,\bar{w}^{\dot{\beta}}{}_v\big)\,\mathrm{e}^{-\mathrm{i}p\cdot x_0}\ . \tag{4.3}$$

where x_0 is the location of the D-instanton inside the world-volume of the D3 branes. Note that all numerical factors and all powers of α' from the various normalizations completely cancel out.

We now turn to the second diagram, represented in Figure 3*b*, which instead depends on the R-R background. It corresponds to the following mixed amplitude

$$\langle\!\langle\, V_{\bar{\mu}}\,\mathcal{V}_{A^I_\mu}(-p)\,V_\mu V_{\mathcal{F}}\,\rangle\!\rangle \tag{4.4}$$

whose evaluation is identical to that of (3.16). Indeed, we find

$$\begin{aligned}\langle\!\langle\, V_{\bar{\mu}}\,\mathcal{V}_{A^I_\mu}(-p)\,V_\mu V_{\mathcal{F}}\,\rangle\!\rangle &= -2\pi^2\,(2\pi\alpha')^{\frac{1}{2}}\,(T^I)^v{}_u\,p^\nu(\bar{\sigma}_{\nu\mu})^{\dot{\alpha}\dot{\beta}}\,\mathcal{F}_{\dot{\alpha}\dot{\beta}}\,\mu^u\bar{\mu}_v\,\mathrm{e}^{-\mathrm{i}p\cdot x_0}\\ &= -\frac{1}{2}\,(T^I)^v{}_u\,p^\nu\,\bar{\eta}^c_{\nu\mu}\,\mu^u\bar{\mu}_v\,C^c\,\mathrm{e}^{-\mathrm{i}p\cdot x_0} \quad ,\end{aligned} \tag{4.5}$$

where in the last step we have introduced the rescaled graviphoton field strength according to (2.23) and (3.15).

There are no other diagrams with only two moduli insertions that contribute to the emission amplitude of the gauge boson. The latter is then given by summing (4.3) and (4.5), namely

$$A^I_\mu(p) = \mathrm{i}\,(T^I)^v{}_u\,p^\nu\,\bar{\eta}^c_{\nu\mu}\Big[(T^c)^u{}_v + (S^c)^u{}_v\Big]\,\mathrm{e}^{-\mathrm{i}p\cdot x_0} \quad , \tag{4.6}$$

where for ease of notation (and for future convenience) we have introduced the $N\times N$ moduli-dependent matrices

$$(T^c)^u_{\ v} = w^u_{\ \dot\alpha}\,(\tau^c)^{\dot\alpha}_{\ \dot\beta}\,\bar w^{\dot\beta}_{\ v} \quad , \quad (S^c)^u_{\ v} = \frac{\mathrm{i}}{2}\,\mu^u\bar\mu_v\,C^c \quad . \tag{4.7}$$

The classical profile of the gauge field in configuration space is obtained by taking the Fourier transform of the emission amplitude (4.6) after inserting a free propagator, that is

$$\begin{aligned} A^I_\mu(x) &= \int \frac{d^4p}{(2\pi)^2}\,A^I_\mu(p)\,\frac{1}{p^2}\,\mathrm{e}^{\mathrm{i}p\cdot x} \\ &= 2\,(T^I)^v_{\ u}\Big[(T^c)^u_{\ v} + (S^c)^u_{\ v}\Big]\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4} \quad . \end{aligned} \tag{4.8}$$

As discussed in [22], this expression represents the leading term in the large distance expansion of the instanton profile. It is important to emphasize that at this stage the field $A^I_\mu(x)$ in (4.8) depends on the *unconstrained* moduli $w^u_{\ \dot\alpha}$, $\bar w^{\dot\beta}_{\ v}$, μ^u and $\bar\mu_v$ of the ADHM construction, but in order to get the dependence from the true moduli, one must enforce the ADHM constraints (3.18) and (3.12). In the undeformed theory, if one imposes the bosonic constraints $W^c = 0$, one finds that the matrices T^c generate a su(2) algebra, while of course the matrices S^c vanish. Thus, choosing a particular solution of the constraints amounts simply to choose a particular embedding of an SU(2) subgroup inside the gauge group U(N). Furthermore, one finds that the gauge field does not have any component along the U(1) factor of U(N). In this way, from (4.8) one retrieves the large distance behaviour of the standard BPST soliton in the singular gauge. In the deformed theory, however, the bosonic ADHM constraints imply that $W^c \neq 0$, and hence these findings are modified.

To see what happens, let us first investigate the algebra of the matrices T and S introduced above. Using their explicit expressions (4.7), it is easy to see that the S's commute among themselves and with the T's, *i.e.*

$$\big[S^a, S^b\big] = 0 \quad , \quad \big[S^a, T^b\big] = 0 \quad . \tag{4.9}$$

Note that to show the second relation, we must use the fermionic constraint (3.12), which for $k=1$ reduces to $w^u_{\ \dot\alpha}\,\bar\mu_u + \varepsilon_{\dot\alpha\dot\beta}\,\mu^u\,\bar w^{\dot\beta}_{\ u} = 0$, actually implying that

$$w^u_{\ \dot\alpha}\,\bar\mu_u = \mu^u\bar w^{\dot\alpha}_{\ u} = 0 \quad . \tag{4.10}$$

The matrices T, with the addition of the matrix

$$\big(T^0\big)^u_{\ v} = w^u_{\ \dot\alpha}\,\bar w^{\dot\alpha}_{\ v} \quad , \tag{4.11}$$

are closed under commutation, and satisfy the algebra

$$\begin{aligned} \big[T^a, T^b\big] &= \mathrm{i}\,\varepsilon^{abc}\,\big(W^0T^c - W^cT^0\big) \quad , \\ \big[T^0, T^a\big] &= -\mathrm{i}\,\varepsilon^{abc}\,W^bT^c \quad , \end{aligned} \tag{4.12}$$

where $W^c \equiv \mathrm{Tr}\big(T^c\big)$ are exactly the quadratic expressions in the w's that appear in the ADHM constraint equations (see (3.10) for $k=1$), and $W^0 \equiv \mathrm{Tr}\big(T^0\big)$. The algebra (4.12) can be recast in the form of a standard u(2) algebra

$$\big[t^a, t^b\big] = \mathrm{i}\varepsilon^{abc}\, t^c \quad , \quad \big[t^0, t^a\big] = 0 \ , \tag{4.13}$$

if we define

$$\begin{aligned} t^a &= \frac{1}{\sqrt{W_0^2 - |\vec{W}|^2}} \big(\mathcal{R}^{-\frac{1}{2}}\big)^{ab} \left(W^0 T^b - W^b T^0\right) \ , \\ t^0 &= \frac{1}{W_0^2 - |\vec{W}|^2} \big(W_0 T^0 - \vec{W} \cdot \vec{T}\big) \ , \end{aligned} \tag{4.14}$$

with $\big(\mathcal{R}\big)^{ab} = W_0^2\, \delta^{ab} - W^a W^b$. These generators are normalized in such a way that $\mathrm{Tr}\big(t^A t^B\big) = \frac{1}{2}\, \delta^{AB}$ for $A = (0, a)$. Inverting the above equations we can express the matrices T appearing in the gauge field profile in terms of the u(2) generators t^a and t^0 as follows

$$T^a = \mathcal{M}^{ab} t^b + W^b t^0 \ , \tag{4.15}$$

where the moduli-dependent matrix $\mathcal{M}$ is

$$\mathcal{M}^{ab} = W^0 \sqrt{W_0^2 - |\vec{W}|^2}\, \big(\mathcal{R}^{-\frac{1}{2}}\big)^{ab} \ . \tag{4.16}$$

From (4.9) and (4.15) it follows that the matrices S commute also with the canonical u(2) generators, *i.e.* $\big[S^a, t^b\big] = 0$ and $[S^a, t^0] = 0$. Using this structure, we can then rewrite the classical solution (4.8) as

$$A_\mu^I(x) = 2\Big(\mathcal{M}^{cb}\, \mathrm{Tr}\big(T^I t^b\big) + W^c \mathrm{Tr}\big(T^I t^0\big) + \mathrm{Tr}\big(T^I S^c\big)\Big)\, \bar{\eta}^c_{\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^4} \ . \tag{4.17}$$

From this result we clearly see that the U(N) instanton gauge field contains a part which is aligned, in color space, along a U(2) subgroup determined by the $4N$ bosonic moduli $w^u{}_{\dot\alpha}$ and $\bar{w}^{\dot\alpha}{}_u$ through the matrices t^b and t^0. Both the non-abelian SU(2) and the abelian U(1) $\subset$ U(2) components are present, in a fashion which is specified by the values of W^b and W^0. Moreover, there is a part of the gauge field along another abelian factor, commuting with the previous U(2), that is determined by the matrices S^c which depend on the fermionic moduli μ^u and $\bar{\mu}_u$. However, to fully specify the instanton profile (including the embedding of the U(2) subgroup into U(N)), it is necessary to take into account the ADHM constraints (3.18) and (3.12). For $k=1$, the bosonic ones are just the following three real equations [6]

$$W^c = -\frac{\mathrm{i}}{2}\Big(M' \!\cdot\! M' - \mu^u \bar{\mu}_u\Big) C^c \equiv \hat{W}^c \ , \tag{4.18}$$

[6]These three constraints reduce the number of independent bosonic moduli to $4N-3$. Moreover, a common phase rotation $w \to \mathrm{e}^{\mathrm{i}\theta} w$, $\bar{w} \to \mathrm{e}^{-\mathrm{i}\theta}\bar{w}$ leaves invariant the matrices t^a, t^0 and their traces. The true bosonic moduli are therefore $4N-4$, corresponding to the $4N-5$ parameters of the coset $U(N)/(U(N-2)\times U(1))$ plus the size of the instanton.

and so all we have to do is simply substitute $W^c = \hat{W}^c$ in the previous formulae and obtain the gauge field profile.

To make contact with the $\mathcal{N}=1/2$ instanton solutions recently obtained in [18, 19], let us choose a specific solution to the bosonic constraints (4.18). Decomposing the index u as $u = (\dot\beta, i)$, with $\dot\beta = 1,2$ and $i = 3,\ldots,N$, we set

$$\begin{cases} w^{\dot\beta}{}_{\dot\alpha} = \rho\,\delta^{\dot\beta}{}_{\dot\alpha} + \frac{1}{4\rho}\,\hat{W}^c\,(\tau^c)^{\dot\beta}{}_{\dot\alpha} \quad , \\ w^{i}{}_{\dot\alpha} = 0 \quad , \end{cases} \tag{4.19}$$

which, in matrix notation, corresponds to choose w as the $N\times 2$ matrix

$$w = \begin{pmatrix} \rho\,\mathbf{1} + \frac{1}{4\rho}\,\hat{W}^c\,\tau^c \\ \mathbf{0}_{(N-2)\times 2} \end{pmatrix} \quad . \tag{4.20}$$

The moduli $\bar{w}^{\dot\alpha}{}_u$ are simply the entries of the hermitian conjugate matrix $w^\dagger$. It is very easy to verify that with this choice $W^c \equiv \mathrm{Tr}(w\tau^c w^\dagger) = \hat{W}^c$ as required; moreover, the parameter ρ (which, for $\hat{W}^c = 0$, represents the size of the instanton) appears in

$$W^0 \equiv \mathrm{Tr}(ww^\dagger) = \frac{1}{2}\Big(4\rho^2 + \frac{1}{4\rho^2}\,|\vec{\hat{W}}|^2\Big) \ . \tag{4.21}$$

Having fixed w and $\bar{w}$ as in (4.20), we can make a specific choice of the fermionic moduli μ and $\bar\mu$ and solve the constraints (4.10) by setting

$$\mu^{\dot\alpha} = \bar\mu_{\dot\alpha} = 0 \quad . \tag{4.22}$$

Furthermore, up to a $\mathrm{U}(N-2)$ rotation, we can choose a single entry of μ^i, say μ^3, to be different from zero. With this specific choice, we therefore have

$$\hat{W}^c = -\frac{\mathrm{i}}{2}\Big(M'{\cdot}M' - \mu^3\bar\mu_3\Big)C^c \quad , \tag{4.23}$$

and hence expressions of degree three or more in $\hat{W}^c$ vanish because of the grassmaniann nature of the parameters μ^3, $\bar\mu_3$ and M'^α. All in all, with this specific solution of the ADHM constraints, the instanton gauge field (4.17) can be easily described by giving its matrix elements $(A_\mu)^u{}_v$ and decomposing the index u as $u = (\dot\alpha, i)$, with $i = 3,\ldots,N$. The result is

$$\begin{aligned}(A_\mu)^{\dot\alpha}{}_{\dot\beta} = &\Big\{\rho^2(\tau^c)^{\dot\alpha}{}_{\dot\beta} - \frac{\mathrm{i}}{4}\,\Big(M'{\cdot}M' - \mu^3\bar\mu_3\Big)C^c\,\delta^{\dot\alpha}{}_{\dot\beta} \\ &- \frac{1}{32\rho^2}\Big(|\vec{C}|^2(\tau^c)^{\dot\alpha}{}_{\dot\beta} - 2C^cC^b(\tau_b)^{\dot\alpha}{}_{\dot\beta}\Big)M'{\cdot}M'\,\mu^3\bar\mu_3\Big\}\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\end{aligned} \tag{4.24}$$

for the components in the upper left block. Moreover, there is also a non-vanishing component outside this block, namely

$$(A_\mu)^3{}_3 = \frac{\mathrm{i}}{2}\,\mu^3\bar\mu_3\,C_c\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4} \quad . \tag{4.25}$$

The above expressions are in agreement with the solution recently found in [19]. In the comparison one has to take into account the different normalizations and conventions, as well as the fact that their solution is in the regular gauge, while ours is in the singular gauge. Furthermore, what we have determined is just the leading term in the long distance expansion $\rho^2/(x-x_0)^2 \ll 1$ of the full instanton solution.

As discussed in [22], mixed disks act as a source also for the gaugino field $\Lambda_\alpha(x)$. In fact they account for the leading term at long distance of the gaugino profile in the super-instanton solution

$$\Lambda^{\alpha,\,I}(x) = -2\mathrm{i}\,(T^I)^v{}_u\big(w^u{}_{\dot\beta}\,\bar\mu_v+\mu^u\,\bar w_{\dot\beta v}\big)(\bar\sigma_\nu)^{\dot\beta\alpha}\frac{(x-x_0)^\nu}{(x-x_0)^4}+\frac{\mathrm{i}}{2}\,M'^{\beta}\,(\sigma^{\mu\nu})_\beta{}^\alpha\,F^I_{\mu\nu}(x) \quad (4.26)$$

where $F^I_{\mu\nu}$ is the gauge field strength. No diagram involving R-R insertions that could correct this result survives in the field theory limit, and thus (4.26) is the gaugino profile at large distance also in the $\mathcal{N}=1/2$ theory. Finally, we recall that with the replacement

$$M'^{\alpha} \longrightarrow M'^{\alpha} - \bar\zeta_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\beta}\,a'_\mu \quad (4.27)$$

in all previous formulas one can account for the superconformal zero-modes of the instanton that are parameterized by $\bar\zeta$.

We conclude by noting that the sub-leading terms in the large distance expansion of the super-instanton solution can be obtained by a perturbative analysis [22] in which more sources (*i.e.* more mixed disks) emit each a gauge boson or a gaugino, which then interact with the (deformed) vertices of the $\mathcal{N}=1/2$ Yang-Mills theory to produce a single gauge boson or gaugino. However, this is exactly the same procedure which has been followed in [18, 19] to determine in a purely field-theoretical framework the (deformed) super-instanton solution, and hence to repeat it here would not add much to our discussion. On the other hand, in the evaluation of instanton-induced or instanton-modified correlators one typically takes into account just the leading contribution in the large-distance expansion of the instanton solution in the singular gauge, which is what the mixed disks provide.

It would be interesting to generalize this analysis to models with extended supersymmetry and to other kinds of closed string backgrounds. It would be nice also to repeat the calculation of the open string scattering amplitudes presented in this paper using the Berkovits formalism [33] which, in contrast to the RNS formalism, allows to treat the R-R background in an exact manner.

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A. Notations and conventions

A.1 Target-space conventions

Indices: We denote by $\mu = 1, 2, 3, 4$ the directions in the 4-dimensional Euclidean world-volume of the D3 branes. By α and $\dot{\alpha}$ we denote, respectively, chiral and anti-chiral spinor indices in the same space. We use $u, v, \ldots = 1, \ldots, N$ to enumerate the D3-branes and $i, j = 1, \ldots, k$ to enumerate the D-instantons. The indices $u, v, \ldots$ transform in the fundamental (or anti-fundamental, depending whether they are in upper or lower position) of the $\mathrm{U}(N)$ gauge group, while the indices $i, j, \ldots$ transform in the (anti)-fundamental of $\mathrm{U}(k)$. We reserve capital indices $I, J, \ldots$ for the adjoint of $\mathrm{U}(N)$.

Gauge fields: We define the non-abelian field strength in terms of a hermitian connection $A_\mu = A_\mu^I \, T^I$ as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i} \left[A_\mu, A_\nu \right] \quad . \tag{A.1}$$

d = 4 Clifford algebra: Let us define the matrices $(\sigma^\mu)_{\alpha\dot{\beta}}$ and $(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}$ with

$$\sigma^\mu = (\mathrm{i}\vec{\tau}, \mathbf{1}) \quad , \quad \bar{\sigma}^\mu = \sigma_\mu^\dagger = (-\mathrm{i}\vec{\tau}, \mathbf{1}) \quad , \tag{A.2}$$

where τ^c are the ordinary Pauli matrices. They satisfy the Clifford algebra

$$\sigma_\mu \bar{\sigma}_\nu + \sigma_\nu \bar{\sigma}_\mu = 2\delta_{\mu\nu} \, \mathbf{1} \quad , \tag{A.3}$$

and correspond to a Weyl representation of the γ-matrices acting on chiral or anti-chiral spinors ψ_α or $\psi^{\dot{\alpha}}$. Out of these matrices, the SO(4) generators are defined by

$$\sigma_{\mu\nu} = \frac{1}{2}(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) \quad , \quad \bar{\sigma}_{\mu\nu} = \frac{1}{2}(\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu) \quad . \tag{A.4}$$

The matrices $\sigma_{\mu\nu}$ are self-dual and thus generate the $\mathrm{SU}(2)_\mathrm{L}$ factor of SO(4); the anti self-dual matrices $\bar{\sigma}_{\mu\nu}$ generate instead the $\mathrm{SU}(2)_\mathrm{R}$ factor. The charge conjugation matrix C is block-diagonal in the Weyl basis, and is given by $C^{\alpha\beta} = -\varepsilon^{\alpha\beta}$ and $C^{\dot{\alpha}\dot{\beta}} = -\varepsilon^{\dot{\alpha}\dot{\beta}}$ with $\varepsilon^{12} = \varepsilon_{12} = -\varepsilon^{\dot{1}\dot{2}} = -\varepsilon_{\dot{1}\dot{2}} = +1$. Moreover we raise and lower spinor indices as follows

$$\psi^\alpha = \varepsilon^{\alpha\beta} \, \psi_\beta \quad , \quad \psi_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}} \, \psi^{\dot{\beta}} \quad . \tag{A.5}$$

The generators $(\sigma^{\mu\nu})_{\alpha\beta}$ and $(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}$, in which the indices have been lowered or raised according to the above rule, are *symmetric* in the spinor indices.

The explicit mapping of a self-dual SO(4) tensor into the adjoint representation of the $\mathrm{SU}(2)_{\mathrm{L}}$ factor is realized by the 't Hooft symbols $\eta^c_{\mu\nu}$; the analogous mapping of an anti self-dual tensor into the adjoint of the $\mathrm{SU}(2)_{\mathrm{R}}$ subgroup is realized by $\bar{\eta}^c_{\mu\nu}$. Specifically we have

$$(\sigma_{\mu\nu})_{\alpha}^{\ \beta} = \mathrm{i}\,\eta^c_{\mu\nu}\,(\tau^c)_{\alpha}^{\ \beta} \quad , \quad (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} = \mathrm{i}\,\bar{\eta}^c_{\mu\nu}\,(\tau^c)^{\dot{\alpha}}_{\ \dot{\beta}} \quad . \tag{A.6}$$

Interpreted as 4×4 matrices, the 't Hooft symbols satisfy the algebra

$$\eta^c\eta^d = -\delta^{cd}\mathbf{1} - \varepsilon^{cde}\eta^e \tag{A.7}$$

with an analogous formula for the $\bar{\eta}$'s. We also have

$$\eta^c_{\mu\nu}\,\eta^{d\,\mu\nu} = 4\,\delta^{cd} \quad , \tag{A.8}$$

$$\eta^c_{\mu\nu}\,\eta^c_{\rho\sigma} = \delta_{\mu\rho}\,\delta_{\nu\sigma} - \delta_{\mu\sigma}\,\delta_{\nu\rho} + \varepsilon_{\mu\nu\rho\sigma} \quad . \tag{A.9}$$

Analogous formulas hold for the $\bar{\eta}$'s with a minus sign in the ε terms of (A.9). From (A.9) and (A.6) it also follows

$$\begin{aligned} \mathrm{tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) &= 2\big(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} + \varepsilon_{\mu\nu\rho\sigma}\big) \quad , \\ \mathrm{tr}\left(\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma}\right) &= 2\big(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} - \varepsilon_{\mu\nu\rho\sigma}\big) \quad , \end{aligned} \tag{A.10}$$

where the trace is over the undotted or dotted spinor indices. Another useful formula is

$$(\tau_c)^{\dot{\alpha}}_{\ \dot{\beta}}(\tau^c)^{\dot{\gamma}}_{\ \dot{\delta}} = \delta^{\dot{\alpha}}_{\ \dot{\delta}}\delta^{\dot{\gamma}}_{\ \dot{\beta}} - \varepsilon^{\dot{\alpha}\dot{\gamma}}\varepsilon_{\dot{\beta}\dot{\delta}} \quad , \tag{A.11}$$

from which, after using (A.6), it follows

$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(\bar{\sigma}_{\mu\nu})^{\dot{\gamma}\dot{\delta}} = -4(\tau_c)^{\dot{\alpha}\dot{\beta}}(\tau^c)^{\dot{\gamma}\dot{\delta}} = 4(\varepsilon^{\dot{\alpha}\dot{\gamma}}\varepsilon^{\dot{\beta}\dot{\delta}} + \varepsilon^{\dot{\alpha}\dot{\delta}}\varepsilon^{\dot{\beta}\dot{\gamma}}) \quad . \tag{A.12}$$

(Anti) self-dual tensors: Any antisymmetric tensor $\mathcal{F}_{\mu\nu}$ decomposes into a self-dual and an anti self-dual component according to $\mathcal{F}_{\mu\nu} = \mathcal{F}^{(+)}_{\mu\nu} + \mathcal{F}^{(-)}_{\mu\nu}$ where

$$\mathcal{F}^{(\pm)}_{\mu\nu} = \pm\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{(\pm)}_{\rho\sigma} \quad . \tag{A.13}$$

We can also write $\mathcal{F}^{(\pm)}_{\mu\nu} = (\mathcal{F}_{\mu\nu} \pm \widetilde{\mathcal{F}}_{\mu\nu})/2$, with $\widetilde{\mathcal{F}}_{\mu\nu} \equiv \varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma}/2$.

Given an anti self-dual tensor $\mathcal{F}^{(-)}_{\mu\nu}$, we can map it to a 3-vector transforming in the adjoint representation of $\mathrm{SU}(2)_{\mathrm{R}}$ using the anti self-dual t'Hooft symbols $\bar{\eta}^c_{\mu\nu}$ according to

$$\mathcal{F}^{(-)}_{\mu\nu} = \mathcal{F}_c\bar{\eta}^c_{\mu\nu} \quad , \quad \mathcal{F}^c = \frac{1}{4}\mathcal{F}^{(-)\mu\nu}\bar{\eta}^c_{\mu\nu} \quad . \tag{A.14}$$

We can organize the three degrees of freedom of the anti self-dual tensor into a *symmetric* dotted bi-spinor by setting

$$\mathcal{F}^{(-)}_{\mu\nu} = \frac{1}{2}\,\mathcal{F}_{\dot\alpha\dot\beta}(\bar\sigma^{\mu\nu})^{\dot\alpha\dot\beta} \quad , \quad \mathcal{F}_{\dot\alpha\dot\beta} = \frac{1}{4}\,\mathcal{F}^{(-)}_{\mu\nu}(\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta} \quad . \tag{A.15}$$

Using (A.6), we can also write

$$\mathcal{F}^{c} = \frac{\mathrm{i}}{2}\,\mathcal{F}_{\dot\alpha\dot\beta}(\tau^c\varepsilon)^{\dot\alpha\dot\beta} \quad , \quad \mathcal{F}_{\dot\alpha\dot\beta} = \mathrm{i}\,\mathcal{F}_{c}(\varepsilon\tau^c)_{\dot\alpha\dot\beta} \quad . \tag{A.16}$$

Given any two anti self-dual tensors $\mathcal{F}^{(-)}_{\mu\nu}$ and $\mathcal{G}^{(-)}_{\mu\nu}$, we can contract them as follows

$$\vec{\mathcal{F}}\cdot\vec{\mathcal{G}} \equiv \mathcal{F}^c\mathcal{G}_c = \frac{1}{4}\,\mathcal{F}^{(-)\mu\nu}\mathcal{G}^{(-)}_{\mu\nu} = \frac{1}{2}\,\mathcal{F}^{\dot\alpha\dot\beta}\mathcal{G}_{\dot\alpha\dot\beta} \quad , \tag{A.17}$$

where in the last step we have used (A.10).

The internal orbifold space: In order to engineer a $\mathcal{N}=1$ gauge theory with D3-branes, we take the six-dimensional transverse space to be an orbifold, obtained by modding out the space $\mathbb{R}^6$ corresponding to the directions $x^5,\ldots,x^{10}$ (and to $\psi^5,\ldots\psi^{10}$) by the action of a $\mathbb{Z}_2\times\mathbb{Z}_2$ group. The two generators g_1 and g_2 of this group act as follows: g_1 is a π rotation in the 7-8 plane and a $-\pi$ rotation in the 9-10 plane; g_2 is a π rotation in the 5-6 plane and a $-\pi$ rotation in the 9-10 plane.

Given the Clifford algebra of the matrices $\gamma^5,\ldots\gamma^{10}$ (which in our stringy perspective are related to the 0-modes of $\psi^5,\ldots\psi^{10}$), one can easily see that the combinations $e^{\pm}_{5-6}=(\gamma^5\pm\mathrm{i}\gamma^6)/2$, $e^{\pm}_{7-8}=(\gamma^7\pm\mathrm{i}\gamma^8)/2$ and $e^{\pm}_{9-10}=(\gamma^9\pm\mathrm{i}\gamma^{10})/2$ are fermionic creation and annihilation operators. Thus, the 8-dimensional spinor space is spanned by the states $|A\rangle=|\pm\frac{1}{2}\rangle_{5,6}\otimes|\pm\frac{1}{2}\rangle_{7,8}\otimes|\pm\frac{1}{2}\rangle_{9,10}$, where $|\pm\frac{1}{2}\rangle_{5,6}$ have eigenvalues $\pm\mathrm{i}/2$ with respect to the the Lorentz generator $J_{56}=[\gamma^5,\gamma^6]/4=\mathrm{i}\sigma^3/2$, and similarly for the 7-8 and 9-10 directions. Then, on the spinor space the two generators of the $\mathbb{Z}_2\times\mathbb{Z}_2$ group are

$$\begin{aligned} g_1 &\to \mathbf{1}\otimes \mathrm{e}^{\pi J_{78}}\otimes \mathrm{e}^{-\pi J_{9,10}} = \mathbf{1}\otimes \mathrm{e}^{\mathrm{i}\frac{\pi}{2}\sigma^3}\otimes \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}\sigma^3} = \mathbf{1}\otimes(\mathrm{i}\sigma^3)\otimes(-\mathrm{i}\sigma^3) \quad , \\ g_2 &\to \mathrm{e}^{\pi J_{56}}\otimes\mathbf{1}\otimes \mathrm{e}^{-\pi J_{9,10}} = \mathrm{e}^{\mathrm{i}\frac{\pi}{2}\sigma^3}\otimes\mathbf{1}\otimes \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}\sigma^3} = (\mathrm{i}\sigma^3)\otimes\mathbf{1}\otimes(-\mathrm{i}\sigma^3) \quad . \end{aligned} \tag{A.18}$$

It is easy to see that the only spinor states which are invariant under g_1 and g_2 are $|+\frac{1}{2}\rangle_{5,6}\otimes|+\frac{1}{2}\rangle_{7,8}\otimes|+\frac{1}{2}\rangle_{9,10}$ and $|-\frac{1}{2}\rangle_{5,6}\otimes|-\frac{1}{2}\rangle_{7,8}\otimes|-\frac{1}{2}\rangle_{9,10}$. In other words, the only surviving spinor weights are

$$\vec\lambda^{(+)} = (+\frac{1}{2},+\frac{1}{2},+\frac{1}{2}) \quad , \quad \vec\lambda^{(-)} = (-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) \quad . \tag{A.19}$$

The first one is chiral, whilst the second is anti-chiral.

A.2 World-sheet conventions

Spin fields and bosonization: As usual, to discuss SO($2N$) spin fields we utilize the Frenkel-Kaç [30] construction (see, for example, [31]). Out of $2N$ world-sheet fermions ψ^m, a SO($2N$) current is defined as $J^{mn} = :\psi^m\psi^n:$. Grouping the directions in pairs, one introduces N world-sheet bosons φ_i, ($i = 1, \ldots N$) by

$$\frac{\psi^{2i-1} \pm \mathrm{i}\, \psi^{2i}}{\sqrt{2}} = c_i\ \mathrm{e}^{\pm \mathrm{i}\, \varphi_i} \quad , \tag{A.20}$$

where c_i are cocycle factors needed to maintain the fermionic statistic. In a Cartan basis with Cartan generators $H_i = J^{2i-1,2i} = :\psi^{2i-1}\psi^{2i}:$, from (A.20) we get

$$H_i = \mathrm{i}\partial\varphi_i \quad . \tag{A.21}$$

Generators associated to a root $\vec{\alpha}$ are represented by $E_{\vec{\alpha}} = \mathrm{e}^{\mathrm{i}\vec{\alpha}\cdot\vec{\varphi}}$; more generally, operators transforming under SO($2N$) as specified by a weight vector $\vec{\lambda}$ are realized as

$$O_{\vec{\lambda}} = c_{\vec{\lambda}}\ \mathrm{e}^{\mathrm{i}\vec{\lambda}\cdot\vec{\varphi}} \quad , \tag{A.22}$$

where again $c_{\vec{\lambda}}$ is a cocycle factor.

Spin fields transform in a spinor representation: S^A is associated to a spinor weight $\vec{\lambda}^A$, with $\lambda_i^A = \pm\frac{1}{2}$ (if the product of all the signs is plus or minus, the spinor is, respectively, chiral or anti-chiral).

Correlators among operators of definite SO($2N$) weights are easily found in the bosonized formulation, since for each boson φ_i we have

$$\Big\langle \prod_k \mathrm{e}^{\mathrm{i}\beta^k\varphi_i}(y_k) \Big\rangle \simeq \delta\big(\sum_k \beta^k\big) \prod_{k<m} (y_k - y_m)^{-\beta_k\beta_m} \quad , \tag{A.23}$$

and other well-known formulae when also $\partial\varphi_i$ operators are inserted.

In deriving the correlators listed below by means of the bosonization formulae, we will not explicitly take into account the cocycle factors, but rather summarize their presence into "effective" rules for the choice of signs and phases.

Spacetime SO(4) correlators: Our bosonization conventions are that the chiral spin fields S^α correspond to the weights $(+\frac{1}{2}, +\frac{1}{2})$ for $\alpha = 1$ and $(-\frac{1}{2}, -\frac{1}{2})$ for $\alpha = 2$. For the anti-chiral spin fields $S^{\dot\alpha}$, instead, $\dot\alpha = 1$ corresponds to $(+\frac{1}{2}, -\frac{1}{2})$ and $\dot\alpha = 2$ to $(-\frac{1}{2}, +\frac{1}{2})$. With these positions, and the general formulae discussed above, one derives the following correlators that have been used in the main text.

The non-vanishing 4-point correlator involving spin fields of different chiralities is

$$\big\langle S_\gamma(y_1) S_\delta(y_2) S^{\dot\alpha}(z) S^{\dot\beta}(\bar z) \big\rangle = \varepsilon_{\gamma\delta}\, \varepsilon^{\dot\alpha\dot\beta}\, (y_1 - y_2)^{-\frac{1}{2}}\, (z - \bar z)^{-\frac{1}{2}} \quad , \tag{A.24}$$

while the correlator with a current and two spin fields is given by

$$\big\langle :\psi_\mu\psi_\nu:(y_3)\, S^{\dot\alpha}(z) S^{\dot\beta}(\bar z) \big\rangle = \frac{1}{2}\, (\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta} (z - \bar z)^{\frac{1}{2}}\, (y_3 - z)^{-1}\, (y_3 - \bar z)^{-1} \quad . \tag{A.25}$$

A similar formula holds for chiral spin fields. A 5-point correlators between one current and four spin fields plays a crucial role in the present paper and is given by

$$
\begin{aligned}
&\big\langle S_\gamma(y_1) S_\delta(y_2) :\!\psi^\mu\psi^\nu\!:(y_3)\, S^{\dot\alpha}(z) S^{\dot\beta}(\bar z)\big\rangle = \frac{1}{2}\,(y_1-y_2)^{-\frac12}(z-\bar z)^{-\frac12} \\
&\quad\times\left((\sigma^{\mu\nu})_{\gamma\delta}\,\varepsilon^{\dot\alpha\dot\beta}\,\frac{(y_1-y_2)}{(y_1-y_3)(y_2-y_3)} + \varepsilon_{\gamma\delta}\,(\bar\sigma^{\mu\nu})^{\dot\alpha\dot\beta}\,\frac{(z-\bar z)}{(y_3-z)(y_3-\bar z)}\right) \quad .
\end{aligned}
\tag{A.26}
$$

Correlators on $\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$: According to (A.19) the only surviving spin fields on the orbifold $\mathbb{R}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ are

$$
S^{(+)} = \mathrm{e}^{\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2+\varphi_3)} \quad , \quad S^{(-)} = \mathrm{e}^{-\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2+\varphi_3)} \quad , \tag{A.27}
$$

up to cocycle factors. There is a single non-vanishing 4-spin correlator, which is crucial in our computations and is given by

$$
\begin{aligned}
&\big\langle S^{(-)}(y_1) S^{(-)}(y_2) S^{(+)}(z) S^{(+)}(\bar z)\big\rangle \\
&\quad = (y_1-y_2)^{\frac34}\,(y_1-z)^{-\frac34}\,(y_1-\bar z)^{-\frac34}\,(y_2-z)^{-\frac34}\,(y_2-\bar z)^{-\frac34}\,(z-\bar z)^{\frac34} \quad .
\end{aligned}
\tag{A.28}
$$

Bosonic twist fields: For the D3/D(–1) and the D(–1)/D3 strings, the fields X^μ along the world-volume of the D3 branes describe Neumann-Dirichlet directions. Their twisted boundary conditions can be seen as due to twist and anti-twist fields Δ and $\bar\Delta$ that change the boundary conditions from Neumann to Dirichlet and vice-versa by introducing a cut in the world-sheet (see for example Ref. [32]). The twist fields Δ and $\bar\Delta$ are bosonic operators with conformal dimension 1/4 and their OPE's are

$$
\Delta(y_1)\,\bar\Delta(y_2) \sim (y_1-y_2)^{-\frac12} \quad , \quad \bar\Delta(y_1)\,\Delta(y_2) \sim -\,(y_1-y_2)^{-\frac12} \quad , \tag{A.29}
$$

where the minus sign in the second correlator is an "effective" rule to correctly account for the space-time statistics in correlation functions. More generally, one can show that

$$
\Big\langle\, \bar\Delta(y_1)\,\mathrm{e}^{-\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X(y_2)}\,\Delta(y_3)\,\Big\rangle = -\,\mathrm{e}^{-\mathrm{i}p\cdot x_0}\,(y_1-y_3)^{-\frac12} \quad , \tag{A.30}
$$

where x_0 denotes the location of the D-instantons inside the world-volume of the D3 branes. This correlator is crucial in computing the profile of the fields emitted by mixed disks, as shown in (4.3) and (4.5).

Superghosts: As usual, we adopt the bosonized treatment of [24] of the superghost system. We use systematically the following correlator between vertices of the type $\mathrm{e}^{-\frac12\phi}$, where ϕ is the chiral boson (with background charge 2) introduced in this formalism, namely

$$
\begin{aligned}
&\big\langle \mathrm{e}^{-\frac12\phi(y_1)}\,\mathrm{e}^{-\frac12\phi(y_2)}\,\mathrm{e}^{-\frac12\phi(z)}\,\mathrm{e}^{-\frac12\phi(\bar z)}\big\rangle \\
&\quad = \Big[(y_1-y_2)\,(y_1-z)\,(y_1-\bar z)\,(y_2-z)\,(y_2-\bar z)\,(z-\bar z)\Big]^{-\frac14} \quad .
\end{aligned}
\tag{A.31}
$$

Conjugation conventions: In the NS sector, the conjugation properties of the polarizations are unambiguously fixed by the expression of the associated vertices themselves. As an example, consider the vertices for the w and $\bar{w}$ moduli, given in (3.6). The conjugate of $\Delta\, S^{\dot\alpha}\,\mathrm{e}^{-\phi}$ is determined by the two-point functions of the involved conformal fields, and is $\bar\Delta\, S_{\dot\alpha}\,\mathrm{e}^{-\phi}$. From this fact, we deduce the following conjugation rule

$$(w^{iu}{}_{\dot\alpha})^* = \bar{w}^{\dot\alpha}{}_{ui}\ , \tag{A.32}$$

or simply $(w_{\dot\alpha})^\dagger = \bar{w}^{\dot\alpha}$ in a $k\times N$ matrix notation for $w_{\dot\alpha}$.

In the R sector, the conjugate of the superghost part $\mathrm{e}^{-\frac{1}{2}\phi}$, which is typically present in the vertex, is $\mathrm{e}^{-\frac{3}{2}\phi}$ due to the background charge of the chiral boson ϕ, and thus we cannot immediately deduce the behaviour of the polarizations by comparing the conjugated vertices. Nevertheless, the space-time character of the conjugated polarization is determined, so that (up to a phase) consistent conjugation rules can be declared. Our rules are the following (in matrix notation w.r.t. to Chan-Paton indices)

$$\begin{aligned} (\Lambda_1)^\dagger &= \mathrm{i}\Lambda_2 \quad , \quad (\Lambda_2)^\dagger = \mathrm{i}\Lambda_1 \quad , \\ (M_1')^\dagger &= \mathrm{i}M_2' \quad , \quad (M_2')^\dagger = \mathrm{i}M_1' \quad , \\ \mu^\dagger &= \mathrm{i}\bar\mu \quad , \quad (\bar\mu)^\dagger = \mathrm{i}\mu \quad . \end{aligned} \tag{A.33}$$

The above relations account for the reality properties of the amplitudes and solutions appearing in the main text.

References

[1] N. Seiberg and E. Witten, JHEP **9909** (1999) 032 [arXiv:hep-th/9908142], and references therein.

[2] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, Phys. Lett. B **574** (2003) 98 [arXiv:hep-th/0302078].

[3] H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **7** (2003) 53 [arXiv:hep-th/0302109].

[4] H. Ooguri and C. Vafa, "Gravity induced C-deformation," [arXiv:hep-th/0303063].

[5] N. Seiberg, JHEP **0306** (2003) 010 [arXiv:hep-th/0305248].

[6] N. Berkovits and N. Seiberg, JHEP **0307** (2003) 010 [arXiv:hep-th/0306226].

[7] S. Ferrara and M. A. Lledo, JHEP **0005** (2000) 008 [arXiv:hep-th/0002084].

[8] D. Klemm, S. Penati and L. Tamassia, Class. Quant. Grav. **20** (2003) 2905 [arXiv:hep-th/0104190].

[9] S. Terashima and J. T. Yee, JHEP **0312** (2003) 053 [arXiv:hep-th/0306237].

[10] R. Britto, B. Feng and S. J. Rey, JHEP **0307** (2003) 067 [arXiv:hep-th/0306215]; R. Britto, B. Feng and S. J. Rey, JHEP **0308** (2003) 001 [arXiv:hep-th/0307091].

[11] M. T. Grisaru, S. Penati and A. Romagnoni, JHEP **0308** (2003) 003 [arXiv:hep-th/0307099]; A. Romagnoni, JHEP **0310** (2003) 016 [arXiv:hep-th/0307209].

[12] T. Araki, K. Ito and A. Ohtsuka, Phys. Lett. B **573** (2003) 209 [arXiv:hep-th/0307076].

[13] R. Britto and B. Feng, Phys. Rev. Lett. **91** (2003) 201601 [arXiv:hep-th/0307165].

[14] O. Lunin and S. J. Rey, JHEP **0309** (2003) 045 [arXiv:hep-th/0307275].

[15] D. Berenstein and S. J. Rey, Phys. Rev. D **68** (2003) 121701 [arXiv:hep-th/0308049].

[16] M. Alishahiha, A. Ghodsi and N. Sadooghi, "One-loop perturbative corrections to non(anti)commutativity parameter of N = 1/2 supersymmetric U(N) gauge theory," [arXiv:hep-th/0309037].

[17] A. Imaanpur, JHEP **0309** (2003) 077 [arXiv:hep-th/0308171]; A. Imaanpur, JHEP **0312** (2003) 009 [arXiv:hep-th/0311137].

[18] P. A. Grassi, R. Ricci and D. Robles-Llana, "Instanton calculations for N = 1/2 super Yang-Mills theory," [arXiv:hep-th/0311155].

[19] R. Britto, B. Feng, O. Lunin and S. J. Rey, "U(N) instantons on N = 1/2 superspace: Exact solution and geometry of moduli space," [arXiv:hep-th/0311275].

[20] S. Ferrara and E. Sokatchev, Phys. Lett. B **579** (2004) 226 [arXiv:hep-th/0308021].

[21] E. Ivanov, O. Lechtenfeld and B. Zupnik, "Nilpotent deformations of N = 2 superspace," [arXiv:hep-th/0308012].

[22] M. Billo, M. Frau, I. Pesando, F. Fucito, A. Lerda and A. Liccardo, JHEP **0302** (2003) 045 [arXiv:hep-th/0211250].

[23] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Nucl. Phys. B **507** (1997) 259 [arXiv:hep-th/9707068]; P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, Nucl. Phys. B **565** (2000) 397 [arXiv:hep-th/9906214].

[24] D. Friedan, E. J. Martinec and S. H. Shenker, Nucl. Phys. B **271** (1986) 93.

[25] P. Di Vecchia, L. Magnea, A. Lerda, R. Russo and R. Marotta, Nucl. Phys. B **469** (1996) 235 [arXiv:hep-th/9601143].

[26] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld and Y. I. Manin, Phys. Lett. A **65** (1978) 185.

[27] N. Dorey, T. J. Hollowood, V. V. Khoze and M. P. Mattis, Phys. Rept. **371** (2002) 231 [arXiv:hep-th/0206063].

[28] J. Polchinski, Phys. Rev. D **50** (1994) 6041 [arXiv:hep-th/9407031].

[29] M. B. Green and M. Gutperle, Nucl. Phys. B **498** (1997) 195 [arXiv:hep-th/9701093]; M. B. Green and M. Gutperle, JHEP **0002** (2000) 014 [arXiv:hep-th/0002011].

[30] I. B. Frenkel and V. G. Kac, Invent. Math. **62** (1980) 23.

[31] V. A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, Nucl. Phys. B **288** (1987) 173.

[32] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B **261** (1985) 678; S. Hamidi and C. Vafa, Nucl. Phys. B **279** (1987) 465.

[33] N. Berkovits, "ICTP lectures on covariant quantization of the superstring," arXiv:hep-th/0209059.